# AMERICAN UNIVERSITY OF BEIRUT 

Mathematics Department
Math 101 - Quiz II
Fall 2010-2011

Name: $\qquad$ ID: $\qquad$

Please circle your section number:

Section 1
F @ 9:00

Section 2
F @ 10:00

Section 3
F @ 11:00

Section 4
F @ 12:00

## Instructions:

1. Write your NAME and AUB ID number above.
2. Solve the problems on the white sheets at the appropriate place.
3. You may use the blank and the back pages of the white sheet to solve or complete the solution of a problem.

| Part I |  | / 20 |
| :---: | :---: | :---: |
| Part II | 1 | /18 |
|  | 2 | /12 |
|  | 3 | /12 |
|  | 4 | /24 |
|  | 5 | /14 |
|  | 6 | /6 |
|  | 7 | /12 |
| Total |  | /118 |

PART I: Fill in the blanks. (20\%)

1. If $f(c) \leq f(x)$ for all $x$ in the domain of $f$, then $f$ is said to have $\qquad$
$\qquad$ at $x=c$.
2. Can a local maximum also be an absolute maximum for a function $f$ ? (Yes or No)
$\qquad$
3. If $f^{\prime}(c)=0$, then $f$ either a local maximum or a local minimum at the interior point $x=c$. (True or False) $\qquad$
4. If $f$ has a local minimum at the interior point $x=c$, then $f^{\prime}(c)=0$. (True or False) $\qquad$
5. If the second derivative $f^{\prime \prime}(x)$ is positive, the curve of $f$ concave $\qquad$ at that point.
6. Does the condition $f^{\prime \prime}(x)=0$ guarantee a point of inflection? (Yes or No)
7. If $f^{\prime}(x)=0$ for all $x$ in an interval $I$, then $f(x)$ $\qquad$ .
8. Suppose $y=f(x)$ and its first derivative $f^{\prime}(x)$ are continuous over $a \leq x \leq b$. If $f(a)$ and $f(b)$ have opposite signs, then according to the Intermediate Value Theorem there is at least one point $c$ satisfying $a \leq c \leq b$ and $f(c)=$ $\qquad$ .
9. Suppose the function $y=f(x)$ has the derivative $f^{\prime}(x)=\frac{(x-1)\left(x^{2}+3\right)}{x}$. Then the critical points of $f$ are $\qquad$ .
10. 

 Is $f(x)$ continuous on $[-3,2]$ ? (Yes or No)
$\qquad$ .

## PART II:

1. a) (6\%)Find horizontal and vertical asymptotes for the curve $y=\frac{x^{2}-2 x-4}{2 x^{2}-2}$.
b) (6\%)Find the value of $c$ that verify the Mean Value Theorem.

$$
f(x)=x^{2}+2 x-1,[0,1]
$$

c) $(6 \%)$ Find the second derivative of $y=\frac{1}{5 x^{2}}-3 x+x^{2}+7$.
2. (12\%) Consider the function $f(x)=\left\{\begin{array}{cc}2 x+1, & -1 \leq x<0 \\ x^{2}, & 0 \leq x<1 \\ 3, & x=1 \\ -2 x+3, & 1<x<2\end{array}\right.$
i. Does $f(1)$ exist?
ii. Does $\lim _{x \rightarrow 1} f(x)$ exist?
iii. Does $\lim _{x \rightarrow 1} f(x)=f(1)$ ?
iv. Is $f(x)$ continuous at $x=1$ ?
3. a) (6\%)Find an equation for the tangent to the curve $y=x^{3}-4 x+1$ at the point $(2,1)$.
b) (6\%)Find the absolute maximum and minimum values (if they exist) of $f(x)=x^{3}-6 x^{2}+2$ over the interval $0 \leq x \leq 2$.
4. $(24 \%)$ Find the derivative of the following:
a) $y=\left(x^{2}-9\right)\left(3 x^{5}+7 x\right)$
b) $y=\frac{x^{2}-1}{3 x+1}$
c) $y=x \cos (5 x-2)$
d) $y=(\csc x+\tan x) \csc x$
e) $y=\frac{\sin x}{x}+\frac{x}{\sin x}$
f) $y=\sqrt{x+\frac{1}{x}}$
5. Let $f(x)=x^{3}-3 x^{2}+2$.
a) (8\%)Find the intervals in which the function $f$ is increasing and decreasing.
b) (6\%)Find the intervals in which the graph of $f$ concave up or concave down.
6. $(6 \%)$ Find the tangent to the curve $\cos (y)+x y-x^{2}=7$ at the point $(1,0)$
7. (12\%)Let $f$ be the function with domain $\left(-\frac{\pi}{2}, \infty\right)$ be given by

$$
f(x)=\left\{\begin{array}{rc}
x^{2}-x+\frac{\sin 3 x}{4 x} & \text { if } x>0 \\
a & \text { if } x=0 \\
b+\frac{\tan x}{x}-\frac{1}{2} & \text { if }-\frac{\pi}{2}<x<0
\end{array}\right.
$$

Calculate:
i. $\quad \lim _{x \rightarrow 0^{+}} f(x)$
ii. $\quad \lim _{x \rightarrow 0^{-}} f(x)$
iii. Find the values of $a$ and $b$ which make $f$ continuous at $x=0$.

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